Please indicate how much time you spent on this assignment.

Problem 1
Prove that if $A$ is decidable, then $A \circ \{0, 1\}^*$ is decidable.

Problem 2
Let $f : \{0, 1\}^* \to \{0, 1\}^*$ be a function that flips all the bits of its input. For example,

$$f(0110) = 1001.$$ 

Prove that the language

$$B = \{\langle D \rangle \mid D \text{ is a DFA that accepts } w \text{ if and only if } D \text{ accepts } f(w)\}$$

is decidable.

Bonus Problem: 1
Prove that the class of Turing-recognizable languages is closed under union.

Bonus Problem: 2
Let $L$ be a language. Prove that $L$ is Turing-recognizable if and only if there exists a decidable language $\hat{L}$ such that

$$L = \{x \mid \text{there is a } y \text{ such that } \langle x, y \rangle \in \hat{L}\}.$$