Please indicate how much time you spent on this assignment.

Problem 1
Prove that if \( A \) is decidable, then \( A \circ \{0,1\}^* \) is decidable.

Problem 2
Let \( f : \{0,1\}^* \rightarrow \{0,1\}^* \) be a function that flips all the bits of its input. For example,
\[
f(0110) = 1001.
\]
Prove that the language
\[
B = \{ \langle D \rangle | D \text{ is a DFA that accepts } w \text{ if and only if } D \text{ accepts } f(w) \}
\]
is decidable.

Bonus Problem: 1
Prove that the class of Turing-recognizable languages is closed under union.

Bonus Problem: 2
Let \( L \) be a language. Prove that \( L \) is Turing-recognizable if and only if there exists a decidable language \( \hat{L} \) such that
\[
L = \{ x | \text{ there is a } y \text{ such that } \langle x, y \rangle \in \hat{L} \}.
\]