Please indicate how much time you spent on this assignment.

Problem 1
Prove that the language

\[ A = \{ \langle M, q \rangle \mid M \text{ is a TM and } q \text{ is a state of } M \text{ that is never used} \} \]

is undecidable. Note that “never used” simply means that \( M \) never enters the state \( q \) on any input.

Problem 2
We have seen that \( EQ_{DFA} \) is decidable and \( EQ_{TM} \) is undecidable. Let \( EQ_{TD} \) be the language

\[ EQ_{TD} = \{ \langle M, D \rangle \mid M \text{ is a TM, } D \text{ is a DFA, and } L(M) = L(D) \} \]

Prove or disprove that \( EQ_{TD} \) decidable.

Problem 3
Let \( \text{FINITE}_{TM} \) be the language

\[ \text{FINITE}_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is finite} \} \]

Show that \( \text{FINITE}_{TM} \) is undecidable by giving a mapping reduction from a language we already know is undecidable to \( \text{FINITE}_{TM} \).

Problem 4
Prove or disprove each of the following statements about the relation \( \leq_m \).

a) \( \leq_m \) is reflexive

b) \( \leq_m \) is symmetric

c) \( \leq_m \) is transitive

A proof of the affirmative should include the reduction function and the Turing machine that computes it. A disproof should include a counterexample and an argument why the example causes the statement to fail.