Problem 0. Document how much time you spend on each of the following problems and cite any resources you received help from.

Problem 1. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA where

\[ Q = \{q_0, q_1, q_2\}, \quad \Sigma = \{x, y\}, \quad F = \{q_0\}, \]

and where $\delta$ is defined by the transition table

\[
\begin{array}{c|cc}
\delta & x & y \\
\hline
q_0 & q_0 & q_1 \\
q_1 & q_0 & q_2 \\
q_2 & q_0 & q_0 \\
\end{array}
\]

(a) Draw the visual representation of $M$.

(b) Describe in your own words what language $M$ recognizes.

Problem 2. Give the formal descriptions of each of the following DFAs (i.e. set of states, transition table, etc.).
Problem 3. Prove that the following languages are regular by giving an example of a DFA or an NFA that recognizes them.

(a) $L_1 = \{xy \mid x, y \in \{0, 1\}^*\}$
(b) $L_2 = \{w11 \mid w \in \{0, 1\}^*\}$
(c) $L_3 = L_1 \cap L_2$

Problem 4. Let $\Sigma = \{a, b\} \cup \{\text{\texttt{N}}, /, *\}$ be the alphabet for this problem where \texttt{N} represents the \texttt{newline} character.

In Java, there are three different types of comments

1. \texttt{/*...*/},
2. \texttt{//...\texttt{N}}, and
3. \texttt/**...*/,

where the third type is a Javadoc comment.

(a) Construct an NFA that recognizes the language of all such comments.

Note that the entire input string needs to be a comment, therefore your NFA should accept strings like \texttt{//abba\texttt{N}} and reject strings like \texttt{ab//abab\texttt{N}} and even \texttt{//aaab} (since the terminating newline is missing). We also point out that \texttt{/*aba*/aaa*/} is a valid string of the first kind, but \texttt{/*aba*/aaa*/} is NOT (because the first \texttt{*/} terminates the comment).

(b) IDEs often need to distinguish between regular comments and special Javadoc comments (e.g. for syntax highlighting). Construct an NFA that only recognizes strings of the first and second kind (i.e. it rejects Javadoc comments).

Note that \texttt{/**} is a valid comment of the first kind but \texttt{/**\textbackslash/*} is a comment of the third kind.

Challenge Problem (Extra Credit). In class, we discussed how to encode the relation $x + y = z$ into a language. Here we will show an alternative approach of encoding this relation. Consider the alphabet defined by

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$
Note that the alphabet $\Sigma_3$ contains all the size 3 columns of 0s and 1s. Therefore a string in $\Sigma_3^*$ yields three rows of 0s and 1s. Consider each of these three rows to be a binary number and let

$$ADD_2 = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$ 

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in ADD_2,$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in ADD_2,$$

since $001 + 011 = 100$. However,

$$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \notin ADD_2,$$

since $01 + 00 \neq 11$.

Prove that $ADD_2$ is regular.