This homework is due Wednesday 10/4 @ 10:30 PM

Please submit your solutions in a zip file including `recursion.ml`, a PDF (written in \LaTeX), and your \LaTeX source to the instructor via email with the subject:

- CSC 208 (17fa) Homework 1 [<Your Full Name>].

**Problem 1 (Quantitative Analysis)** Let the propositions:

\[ A(x) = x \text{ has money} \]
\[ B(x, y) = x \text{ wants to buy } y's \text{ candy} \]
\[ C(x, y) = x \text{ owes } y \text{ money.} \]

For each of the given formal propositions, translate them into English sentences capturing the intent of the proposition. You are not allowed to use the words “for all” and “exists” in your sentences.

(a) \( \forall x. \exists y. B(x, y) \)
(b) \( \exists x. \forall y. C(x, y) \)
(c) \( \forall x. \exists y. A(x) \rightarrow B(x, y) \)
(d) \( \forall x. \exists y. \forall z. A(x) \rightarrow C(x, y) \rightarrow B(y, z) \)
(e) \( \exists x. \forall y. \exists z. C(x, y) \rightarrow B(y, z) \)

**Problem 2 (Recursion Practice)** Implement each of the following recursive functions in `recursion.ml`.

(a) \( \text{drop} : n \rightarrow 'a \text{ list } \rightarrow 'a \text{ list} \)
(b) \( \text{replicate} : \text{int } \rightarrow 'a \rightarrow 'a \text{ list} \)
(c) \( \text{snoc} : 'a \rightarrow 'a \text{ list } \rightarrow 'a \text{ list} \)
(d) \( \text{zip} : 'a \text{ list } \rightarrow 'b \text{ list } \rightarrow ('a \ast 'b) \text{ list} \)
(e) \( \text{rev} : 'a \text{ list } \rightarrow 'a \text{ list} \)
(f) \( \text{dedup} : 'a \text{ list } \rightarrow 'a \text{ list} \)
(g) \( \text{concat} : ('a \text{ list}) \text{ list } \rightarrow 'a \text{ list} \)
(h) \( \text{split} : \text{int } \rightarrow 'a \text{ list } \rightarrow 'a \text{ list } \ast 'a \text{ list} \)
(i) \( \text{prefixes} : 'a \text{ list } \rightarrow ('a \text{ list}) \text{ list} \)

**Problem 3 (Inductive Hypotheses)** For functions (b), (e), and (g) above, state precisely the induction hypothesis you may assume in the recursive case when proving properties of these functions.

**Problem 4 (Inductive Fallacies)** Consider the following claim:

**Claim 1.** For all lists \( l \) and any two elements \( x1 \) and \( x2, x1 = x2. \)

And the following proof of this claim:

**Proof.** Proof by induction on \( l. \)
• $l = \[]$. There are no elements in $l$ so our claim trivially holds.

• $l = x :: xs$. By our induction hypothesis, we know that for any list smaller than $l$, any two elements of that list are equal. Consider the following two lists:
  
  – $l'$ is the list obtained by removing the first element of $xs$ and replacing it with $x$.
  – $l''$ is the list obtained by removing the second element of $xs$ and replacing it with $x$.

$l'$ and $l''$ are both smaller than $l$ so our induction hypothesis applies and thus all elements within $l'$ are equal, and all elements within $l''$ are equal. However, we know that $x$ is in both $l'$ and $l''$, so we know that all the elements of both lists, and thus $l$, must be equal because they are equal to $x$.

This claim is false. Because it is false, then this alleged proof must also be false. Describe in a few sentences what is the reasoning error made in this proof.

Problem 5 (Induction Parade) For functions (a), (c) and (h), prove the following claims using induction:

Claim 2. $\forall n. l. n \geq 0 \rightarrow \text{length (drop n l)} = \text{length l} - n$.

Claim 3. $\forall v. l. \text{length (snoc v l)} = \text{length l} + 1$.

Claim 4. $\forall n. l. n \geq 0 \rightarrow l1 @ l2 = l$ where $l1 = \text{fst (split n l)}$ and $l2 = \text{snd (split n l)}$.

Problem 6 (Strike Your Own Claim) Write down a property that implies the partial correctness of the prefixes function from problem 2 and prove this claim.