Teaching Induction with Functional Programming and a Proof Assistant

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Why?
Recursive functions and definitions are deeply intertwined with inductive reasoning. Can we use functional programming to better teach induction by revealing the structure of proofs and giving inductive proof a practical purpose for the working programmer?

Introducing InductFun!
InductFun! is an online tutorial system for learning proof by induction in the context of functional programming. Students write programs in a core functional programming language and prove simple properties about them with the aid of a proof assistant.

The InductFun! System

```
type nat =  
  | 0  
  | S nat

let plus (n1:nat) (n2:nat) : nat =  
  match n1 with  
  | 0  -> n2  
  | S n1' -> S (plus n1' n2)
```

Paper-like proof script is translated to and checked by the Coq proof assistant on the server. Its step-by-step output is given back to the student as feedback.

Recursive Functions
Inductive Reasoning
Functional Programs
Simple Inductive Proofs

Lemma plusO : forall (n:nat), plus n 0 = n
Proof
  assume n:nat
  by induction on n:
  case n = 0 =>
      simplify to 0 = 0
      true by reflexivity
  case n = S n' with IHn' : plus n' 0 = n' =>
      simplify to S (plus n' 0) = S n'
      rewrite with IHn' to S n' = S n'
      true by reflexivity
Qed

Pedagogical Goals
For beginning undergraduate CS majors:
1. Teach the mechanics of inductive proofs.
2. Bridge formal and informal reasoning.

Tutorial Outline
1. Functional programming in InductFun!
2. Basic (non-inductive) proofs in InductFun!
3. Inductive proofs!
4. Adv. inductive pfs. (e.g. strengthening).
5. The formal/informal reasoning spectrum.
6. More examples!

Next Steps
• Many UI/language usability tweaks left!
• Trials this Fall → deployment in discrete mathematics course next Spring.
• Working with others interested in using InductFun! In their classrooms.
• Long term goal: automatic feedback and guidance for inductive proofs.

Try It Out!
http://inductfun.org
Interest? Feedback?
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